§ 7.1 QUESTION NUMBER 18

Solution

$$\int \frac{1}{\sqrt{4-x}}\, dx$$

this question will
require the use of sustitution.

let $w = 4 - x$

$$dw = -dx$$

$$dx = -dw$$

$$-\int \frac{1}{\sqrt{w}}\, dw = -\int w^{-\frac{1}{2}}\, dw$$

$$= -2\, w^{\frac{1}{2}}\, dw$$

then substituting for w we have :

$$= -2\sqrt{4-x} + C$$

Major Faults

A few slipped on the use of the negative sign and adding the integration constant to the answer. Solutions did not have the integration constant c added - indefinite integration.

Some students struggled with the choice of what should be substituted , handling the dividion by 1 / 2 which gives a 2 and the integration of $-\int \frac{1}{\sqrt{w}}\, dw$

Only a handful did not know the approach to this question was by substitution.

§ 7.1 QUESTION NUMBER 25

Solution

$$\int \sin^6(5\theta)\cos(5\theta)\,d\theta$$

this question also requires
that we substitute and integrate;

let $w = \sin(5\theta)$

then $dw = 5\cos(5\theta)\,d\theta$

$$1/5\,dw = \cos(5\theta)\,d\theta$$

$$\int \sin^6(5\theta)\cos(5\theta)\,d\theta = \frac{1}{5}\int w^6\,dw$$

$$= \frac{1}{5}\left(\frac{w^7}{7}\right) + c$$

$$= \frac{1}{35}\sin^7(5\theta) + c$$

Major Faults

The general problem on this question was the struggle to choose which part of the integrand to substitute and plugging back in after differentiating.

A few did not see the question required substitution and also did not add the integrating constant to the final answer.

Some had right answers but showed no work as to how they got their answers and others guessed the answers.

§ 7.1 QUESTION NUMBER 34

Solution

$$\int \frac{x+1}{(x^2+2x+19)}\,dx$$

let $w = x^2 + 2x + 19$

$dw = 2x + 2\,dx$

$= 2(x+1)\,dx$

then

$$(x+1)\,dx = \tfrac{1}{2}\,dw$$

substituting the integrand we have

$$\int \frac{x+1}{(x^2+2x+19)}\,dx = \int \frac{1}{2w}\,dw$$

$$= \frac{1}{2}\int \frac{1}{w}\,dw$$

$$= \frac{1}{2}\ln|w| + c$$

$$= \frac{1}{2}\ln\left|x^2+2x+19\right| + c$$

Major Faults

A few solutions did not have the integration constant c added (indefinite integration).

A handful also forgot that the quiz required substitution to solve and ended up using integration by parts.

Most students had a problem substituting especially dealing with the fraction.

§ 7.1 QUESTION NUMBER 66

Solution

$$\int_1^2 \frac{\sin t}{t}\, dt$$

Since the integrand cannot be integrated by substitution, we use a numerical method to have an approximate solution. We choose intervals of 4, n = 4 a = 1 and b = 2.

$$\Delta t = \frac{2-1}{4} = \frac{1}{4}$$

and our left sum will be :

$$= [f(1) + f(5/4) + f(6/4) + f(7/4)]\,\Delta t$$

$$= [\sin 1 + 4/5 \sin(5/4) + 2/3 \sin(3/2) + 4/7 \sin(7/4)]\,(1/4)$$

$$= 0.7070$$

right sum :

$$= [f(5/4) + f(6/4) + f(7/4) + f(8/4)]\,\Delta t$$

$$= [4/5 \sin(5/4) + 2/3 \sin(3/2) + 4/7 \sin(7/4) + 1/2 \sin 2]\,(1/4)$$

= 0.6103

this implies that an approximate value of the integration lies between 0.6103 and 0.7070

$$0.6103 < \int_1^2 \frac{\sin t}{t}\, dt < 0.7070$$

a larger choice of n will give a better estimate of this value.

Major Faults

The general problem on this question was that students did not recognise that the function could not be integrated by substitution. Always be sure to read the instructions as they indicated substitution may not work for all of the problems.

Those who got the hint could not properly use the numerical method (i.e., left and right - hand Riemann sums) to find an approximate solution.